Optimization of the Breakthrough Accuracy in Tunneling Surveys

DR. ADAM CHRZANOWSKI
Department of Surveying Engineering
University of New Brunswick
Fredericton, N.B., Canada

Analysis and optimization of the breakthrough accuracy and a proper design of the underground control survey are the most important tasks of the survey engineer in charge of a tunneling project. The breakthrough point P should be treated as two different points P' and P" in the accuracy analysis, and the breakthrough error is found from the relative error ellipses calculated for P' and P". The accuracy of the underground control surveys (usually open-ended traverses) is much more critical in tunneling surveys than the accuracy of the surface control network. Use of gyrotheodolites decreases the error propagation in underground traverses, particularly when the influence of atmospheric refraction is suspected.

L'analyse, l'obtention de la précision optimale à la jonction et les levés souterrains constituent les tâches les plus importantes des ingénieurs responsables d'un projet de creusage de tunnel. Dans une analyse précise, le point de jonction P' et P'' et la marge d'erreur du point de jonction est évaluée d'après l'erreur relative des ellipses calculées pour P' et P''. Pour les tunnels, la précision des levés souterrains (il s'agit habituellement d'une galerie transversale ouverte) comporte beaucoup plus que la précision du réseau de contrôle de surface. L'utilisation de gyrothéodolites diminue le risque d'erreurs dans les galeries souterraines surtout si l'on croit que la réfraction atmosphérique peut produire certains effets.

Introduction

Construction of long tunnels, whether for railway, highway or hydro projects, is a major engineering undertaking. It is an expensive operation costing between \$10 000 000 and \$20 000 000 a km depending on the diameter of the tunnel, the type of rock and the general hydrogeological conditions.

Modern tunneling techniques allow for a full-face boring of tunnels with diameters of 10 m and more at a speed of 5 m to 20 m a day, followed almost simultaneously by semiautomatic lining of the tunnel excavation with prefabricated concrete or steel rings.

> A number of long tunnels are under consideration in Canada. Probably the most important of these are tunnels that may connect two of Canada's important islands to the mainland. The Strait of Belle Isle Tunnel connecting Newfoundland to Labrador, if built, would be about 18 km long. There are various proposed routes to join Vancouver Island to the mainland. Some of the proposals involve tunnels up to 15 km in length. —*Editor.*

(Pages 5 to 16)

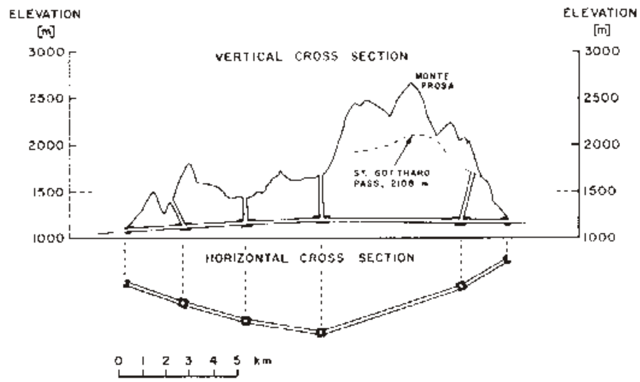

Fig. 1. St. Gotthard highway tunnel in Switzerland.

Long tunnels require intermediate openings to the surface, such as vertical shafts or adits, for ventilation and transportation purposes during construction. As an example, Figure 1 shows longitudinal cross sections of the recently finished St. Gotthard highway tunnel in Switzerland. The tunnel is 16.3 km long with two vertical and two inclined ventilation shafts.

Tunnels are usually driven simultaneously from two opposite entrances (Figure 2). Sometimes, the construction progresses with several headings being worked simultaneously from the intermediate openings which are made in advance.

The high cost of tunneling and recent advances in the technology require a survey engineer to design the survey control for the alignment of the tunnel axis with the highest possible accuracy so that the opposing headings meet at the breakthrough points without any need for an adjustment of the excavations. Generally, an accuracy of 10 mm to 20 mm per km of driven tunnel is required for the meeting of opposite headings. Sometimes even higher accuracy is required as, for instance, in the 14-km-long Arlberg tunnel in Austria, where the survey specifications called for the breakthrough error to be smaller than ± 50 mm [*Posch* 1980] due to a new tunneling method employed in the project.

Many other tasks are required of the survey engineer during the tunnel construction, such as setting-out the specified grade and line of the tunnel, checking profiles of the cross sections of the excavations, guiding the boring machine, or setting out drill holes for blasting operations and, finally, measuring deformations of the tunnel cross sections for safety and maintenance purposes. However, optimization of the breakthrough accuracy and the design of the control survey are the major and most important tasks that must be done before tunnel construction is started.

There are many excellent publications, for example *Wassermann* [1967], dealing with setting-out surveys during tunnel construction. However, there are very few publications that give the proper way of calculating and optimizing breakthrough accuracy. One of the best papers on the subject is by *Rinner* [1976]. Most of the other publications, including even recent textbooks on engineering surveys, give only approximate methods for the prediction of the breakthrough error or, even worse, incomplete and confusing theories.

This paper is presented to clarify the subject of the calculation of the breakthrough error and to discuss some problems dealing with the design of underground control surveys.

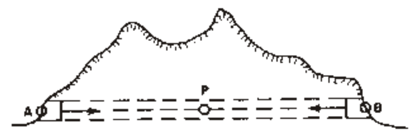

Fig. 2. Tunneling with two opposing headings.

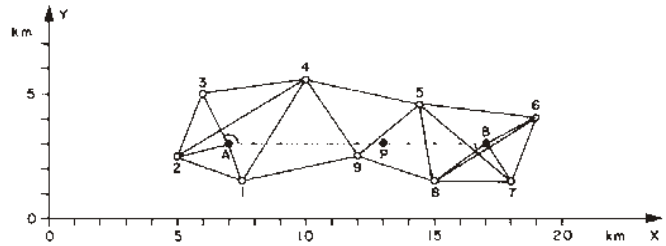

Fig. 3. Horizontal control network for a tunnel construction.

Calculation of the breakthrough error

Only horizontal breakthrough errors will be discussed, but the basic principle given below applies, of course, to the vertical breakthrough as well, and the principle can easily be expanded to a three-dimensional case.

Let us consider the simplest breakthrough case, which is when a straight tunnel is driven simultaneously from two opposite directions (Figure 2). The relative positions (coordinates) of the entrance points A and B must, of course, be known beforehand by connecting them on the surface by a survey network in a local coordinate system as shown, for example in Figure 3.

The axes of the heading A→P and B→P are supposed to meet in the breakthrough point P. The location (coordinates) of the point P are predetermined in accordance with the engineering design of the tunnel construction. Because of unavoidable errors of geodetic measurements in the surface network and errors of the control surveys in the tunnel, the physical location P′ of point P set out by the survey A→P differs from the location P″ set out by the survey B→P. Therefore, in the error analysis, the breakthrough point P should always be treated as two different points P′ and P″, though both have the same design coordinates.

To avoid some numerical problems when using a computer program for the error analysis, the approximate coordinates of points P′ and P″ should be given with slightly different X- and Y-coordinates, which vary by, say, 0.5 m.

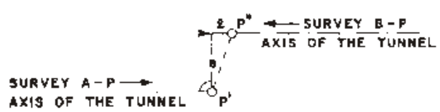

Fig. 4. Breakthrough error.

The distance P′P″ (Figure 4) is the total breakthrough error. The lateral component e of the total breakthrough error is of a much greater importance than the longitudinal component ℓ and, therefore, the accuracy optimization of the breakthrough survey is concerned mainly with the prediction of the value of e.

The total breakthrough error may be defined as a relative positional error for points P′ and P″. It is best described by a relative standard-error curve (pedal curve) for points P′ and P″ (Figure 5) which allows components of the relative positional error to be determined in any desired direction, thus, also in the direction of e.

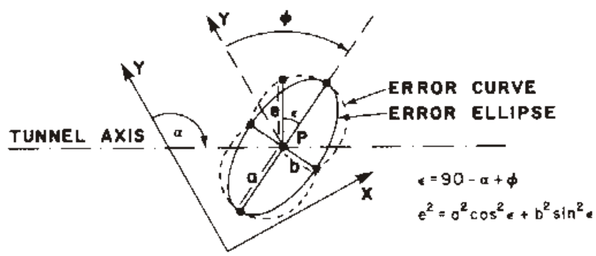

Fig. 5. Relative error ellipse and relative error curve for the breakthrough points P' and P''.

To obtain the error curve, the full variance-covariance matrix Σ_{x_p} for points P′ and P″ must first be calculated using a rigorous propagation of estimated errors of measurements in the underground and surface survey networks which connect points P′ and P″.

The propagation of errors may be calculated by using the well known parametric method (method of observation equations) of the least squares adjustment, which gives the full variance-covariance matrix Σ_x for all coordinates of the nonfixed points in the surface and underground networks. It is assumed that the reader is familiar with the basic principles of error propagation and, therefore, only a brief review of the calculation formula follows. The variance-covariance matrix is calculated from

$$\Sigma_x = \sigma_0^2(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1} \tag{1}$$

where σ_0 is an *a priori* estimated standard deviation of the observation with the unit weight,

A is a matrix of coefficients of observation equations, and

P is the weight matrix of observations.

Usually, the surface control network is established and calculated in advance before the final design of the tunnel is made. Therefore, the variance-covariance matrix Σ_{x_s} for the surface network is known before the breakthrough analysis is performed. In that case, instead of taking all the observations of the surface network into (1), the previously adjusted coordinates of the surface network may be treated as pseudo-observables with the weight matrix

$$\mathbf{P}_{x_s} = \Sigma_{x_s}^{-1} \tag{2}$$

This, in combination with the underground observations, leads to the same result as (1) but in the form

$$\Sigma_x = \sigma_0^2\left[\mathbf{P}_{x_s} + (\mathbf{A}^T\mathbf{P}\mathbf{A})_u\right]^{-1} \tag{3}$$

The matrix $\mathbf{P}_{x_s}$ is only calculated for those surface points to which the underground control network is directly connected and the matrix $(\mathbf{A}^T\mathbf{P}\mathbf{A})_u$ is only calculated for the underground observations that connect the breakthrough point with the surface network.

The final result for the Σ_x matrix is, of course, the same whether the calculations are made using (1) or (3). From Σ_x can be extracted the submatrix Σ_{x_p} for points P′ and P″ in the form

$$\Sigma_{x_p} = \begin{pmatrix} \sigma^2_{x'} & \sigma_{x'y'} & \sigma_{x'x''} & \sigma_{x'y''} \\ \sigma_{x'y'} & \sigma^2_{y'} & \sigma_{y'x''} & \sigma_{y'y''} \\ \sigma_{x'x''} & \sigma_{y'x''} & \sigma^2_{x''} & \sigma_{x''y''} \\ \sigma_{x'y''} & \sigma_{y'y''} & \sigma_{x''y''} & \sigma^2_{y''} \end{pmatrix} \tag{4}$$

and the semimajor and semiminor axes of the relative standard error ellipse can be calculated for points P′ and P″ from

$$a^2 = \frac{1}{2}\left(\sigma^2_{\Delta x} + \sigma^2_{\Delta y} + \sqrt{(\sigma^2_{\Delta x} - \sigma^2_{\Delta y})^2 + 4\sigma^2_{\Delta x\Delta y}} \right) \tag{5}$$

$$b^2 = \frac{1}{2}\left(\sigma^2_{\Delta x} + \sigma^2_{\Delta y} - \sqrt{(\sigma^2_{\Delta x} - \sigma^2_{\Delta y})^2 + 4\sigma_{\Delta x\Delta y}} \right) \tag{6}$$

where

$$\sigma^2{}_{\Delta x} = \sigma^2_{x''} + \sigma^2_{x'} - 2\sigma_{x'x''}$$

$$\sigma^2_{\Delta y} = \sigma^2_{y''} + \sigma^2_{y'} - 2\sigma_{y'y''}$$

$$\sigma_{\Delta x\Delta y} = \sigma_{x''y''} - \sigma_{x''y'} - \sigma_{x'y''} + \sigma_{x'y'}$$

The azimuth ϕ of the semimajor axis a is calculated from

$$\tan 2\phi = \frac{2\sigma_{\Delta x\Delta y}}{\sigma^2_{\Delta x} - \sigma^2_{\Delta y}} \tag{7}$$

The semimajor axis a of the relative error ellipse corresponds to the maximum expected standard deviation σ_{max} of the relative position of P′P″. Standard deviations in any other direction at any angle ϵ from the axis a are described by the error curve and are calculated from

$$\sigma^2_\epsilon = a^2 \cos^2\epsilon + b^2 \sin^2 \epsilon \tag{8}$$

The breakthrough error e is calculated as a component of the error curve in the direction perpendicular to the axis of the tunnel, placing in (8)

$$\epsilon = 90° - \alpha + \phi \tag{9}$$

where α is the azimuth of the axis of the tunnel at P.

The calculated value of e corresponds to a standard deviation of the relative position of P′ and P″ in the direction of e.

Usually, the tolerance limit e_{max} for the breakthrough error is taken as the error at 95 percent probability level. Thus one can write

$$e_{max} = e_{95\%} = 1.96e \qquad \text{or} \qquad e_{max} \simeq 2e \tag{10}$$

The above calculation procedure allows the value of e_{max} to be predicted if the geometrical design of the surface and underground control networks is given and if the type and accuracy of measurements in the networks are known in advance. If the given data leads to a value of e_{max} larger than the tolerance limit established by the constructor, then the design of the control surveys must be changed. This can be done by changing either the components of the **A** matrix, which requires changing the geometry of the networks and type of observations, or by changing the weight matrix **P**, which involves changing the *a priori* estimated accuracy of the measurements. This process, known as an optimization of the survey networks, should be done separately for the surface and underground networks because both networks differ distinctly from each other in geometrical design, flexibility for changes, and even type of measurements. Often the surface network is established by a different contractor than the underground control and, sometimes, the surface network is established before the final design of the breakthrough points is ready. Therefore, as a rule, the breakthrough error is calculated as the sum of two separate influences: the influences e_s of the errors of the surface network and the influence e_u of the underground network. Thus

$$e^2 = e_s^2 + e_u^2 \tag{11}$$

and, going back to (4),

$$\Sigma_{x_p} = (\Sigma_{x_p})_s + (\Sigma_{x_p})_u \tag{12}$$

To calculate the variance-covariance matrix $(\Sigma_{x_p})_s$ for the breakthrough point P, the underground observations are included in the error analysis as errorless. Practically, the underground network, which is a combination of open traverses, can simply be replaced by two errorless simulated distances: $\overline{AP}$ and $\overline{BP}$ (Figure 3), and by two errorless simulated angles, for instance, angles ∢ 3AP and ∢ 6BP which are taken into matrices **A** and **P** of (1) instead of taking all the underground observations. This saves computer time. To avoid some numerical problems with infinite weights for the errorless observations, it is better to attach some negligible small errors to the 'errorless' observations, say 0.1 mm for the distances and 0.01″ for the angles.

To calculate only the influence of the underground networks, which is expressed by the variance-covariance matrix $(\Sigma_{x_p})_u$, all the underground and surface connected observations are taken into (1) with their proper weights, but all the surface points of the control network are taken as fixed and errorless.

A typical, though simplified example is given below for a tunnel 10 km long as shown in Figure 3. The surface network is both triangulation and trilateration (triangulateration) with all directions and all distances measured. The network is adjusted with minimum constraints holding point 5 and azimuth 5-4 fixed in a local coordinate system. Here it should be noted that the choice of minimum constraints is arbitrary and their change does not affect the breakthrough accuracy. The underground network in the example consists of two open traverses 6 km and 4 km long which are supposed to meet at P. The traverse legs are equal to 1 km.

The following accuracies of measurements have been accepted in the example

Distances (surface and underground),	$\sigma_s^2 = (5\text{ mm})^2 + (3\times 10^{-6}S)^2$
Directions (surface network)	$\sigma_s = 1.0''$
Angles (underground traverses)	$\sigma_\beta = 1.2''$

The described analysis gives the following results of the breakthrough error:

— influence of the surface network at the 95 percent probability level:
$e_s = 55$ mm

— influence of the underground traverses at the 95 percent probability level:
$e_u = 125$ mm

The total value of e_{max} at the 95 percent probability level is therefore equal to

$$e_{max} = \sqrt{e_s^2 + e_u^2} = 137 \text{ mm}$$

As can be seen, the accuracy of the underground traverses in the above example is much more critical than the accuracy of the surface network. If the surface network consisted only of a traverse A-1-9-8-B with all directions and distances measured with the above specified accuracy, the value of e_s would increase from 55 mm to 82 mm, but the total value of e_{max} would only increase from 137 mm to 149 mm, or less than 10 percent.

The above result is typical for most practical cases of tunneling surveys. For instance, *Rinner* [1976] describes several actual tunneling projects and gives the accuracy analysis of their breakthrough errors. In each case, the influence of the errors of the surface network is almost negligible in comparison with the influence of the underground traverses.

The following conclusion can be drawn: the surface network is usually overdesigned and its accuracy requirements exaggerated while the underground control is underestimated. Therefore, the optimization of the breakthrough accuracy must concentrate on the optimization and improvement in the design of the underground control surveys which commonly consist of open-ended traverses.

Error propagation in underground control surveys

The elongated shape and comparatively small diameter of tunnels do not provide much choice and flexibility in the design of the underground control network. The network must be established in the form of open-ended traverses, which is most unfavorable from the point of view of the error propagation.

If the tunnel is constructed simultaneously with the intermediate ventilation adits or shafts as in Figure 1, then a check of the underground traverse is possible by connecting it to the surface control network by using, for instance, shaft plumbing techniques that are discussed, among others, in *Chrzanowski et al* [1967] and *Wassermann* [1967]. Nevertheless, in many cases the open traverse has to be run for several kilometres without any self-control. As a check, two or even three orders of traverses are established. First, the lowest order traverse is run simultaneously with the progress of the tunnel. It provides continuous control of the construction operations. The 'construction traverse' usually has to be established with very short legs (of the order of 100 m between stations) due to construction obstacles and the dusty atmosphere at the heading. Once the tunnel has progressed about one to two kilometres, the higher order traverse of a higher accuracy is established which checks the location of the last point of the construction traverse and, if necessary, a correction is introduced to the location or to the coordinates of that point. The procedure is sometimes repeated once more with a third traverse with distances as long as possible and, from all three, the final correction to the axis of the tunnel is calculated. The correction should be calculated as a weighted mean from the results of all the traverses rather than treating the highest order traverse as errorless and applying the correction as equal to the total difference between the results of the higher or lower order traverses. This is particularly important when the influence of some systematic errors such as atmospheric refraction is suspected.

There is not much choice available in the type of measurements that are used in open traverses. All distances and all directions must be measured. If the tunnel is straight then errors of the distance measurements do not affect the lateral breakthrough accuracy. Usually short range EDM instruments are used. The only choice in the optimization process is to decide whether to measure angles only or gyro azimuths or a combination of both.

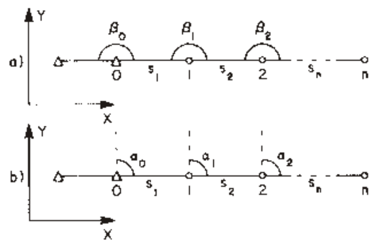

Fig. 6. Open traverse a) with angles
b) with gyro azimuths.

If angles β and distances S are measured, then the variances and covariances of the last point of the open traverse (Figure 6) can be calculated from the well known [*Chrzanowski* 1977] error propagation formulas.

$$\sigma^2_{Y_n} = \sigma^2_\beta \sum_{i=1}^{n} (X_n - X_{i-1})^2 + \sigma^2_S \sum_{i=1}^{n} \left(\frac{Y_i - Y_{i-1}}{S_i} \right)^2 \tag{13}$$

$$\sigma^2_{X_n} = \sigma^2_\beta \sum_{i=1}^{n} (Y_n - Y_{i-1})^2 + \sigma^2_S \sum_{i=1}^{n} \left(\frac{X_i - X_{i-1}}{S_i} \right)^2 \tag{14}$$

$$\sigma_{XY_n} = -\sigma^2_\beta \sum_{i=1}^{n} (Y_n - Y_{i-1})(X_n - X_{i-1}) + \sigma^2_S \sum_{i=1}^{n} \frac{(Y_i - Y_{i-1})(X_i - X_{i-1})}{S_i^2} \tag{15}$$

where σ_β and σ_S are estimated standard deviations of angles and distances respectively.

If gyro azimuths at each station and distances are measured then

$$\sigma^2_{Y_n} = \sigma^2_\alpha \sum_{i=1}^{n} (X_i - X_{i-1})^2 + \sigma^2_S \sum_{i=1}^{n} \left(\frac{Y_i - Y_{i-1}}{S_i} \right)^2 \tag{16}$$

$$\sigma^2_{X_n} = \sigma^2_\alpha \sum_{i=1}^{n} (Y_i - Y_{i-1})^2 + \sigma^2_S \sum_{i=1}^{n} \left(\frac{X_i - X_{i-1}}{S_i} \right)^2 \tag{17}$$

$$\sigma_{XY_n} = -\sigma^2_\alpha \sum_{i=1}^{n} (X_i - X_{i-1})(Y_i - Y_{i-1}) + \sigma^2_S \sum_{i=1}^{n} \frac{(X_i - X_{i-1})(Y_i - Y_{i-1})}{S_i^2} \tag{18}$$

where σ_α is the estimated standard deviation of the gyro measurements.

If a combination of angles and gyro azimuths is used with redundant observations then the general propagation of variances and covariances is applied using, for instance, (1).

In a straight tunnel with its axis parallel to the X-axis of the local coordinate system only σ_{Y_n} will contribute to the lateral breakthrough error. A comparison of (13) and (16) shows immediately that the gyro azimuths measurements give more favorable error propagation than angles. If all the distances in the traverse are approximately the same then to obtain the same value of σ_{Y_n} from (13) and (16) the ratio $\sigma_\alpha/\sigma_\beta$ should be equal to

$$\frac{\sigma_\alpha}{\sigma_\beta} = \sqrt{\frac{\sum_{i=1}^{n} i^2}{n}} = \sqrt{\frac{(n+1)(2n+1)}{6}} \tag{19}$$

where n is the number of the traverse legs.

Equation 19 was derived by substituting $X_n - X_o = n.S$, $X_i - X_{i-1} = S$ etc. in (13) and (16) and by using the rule of the sum of powers of the natural numbers.

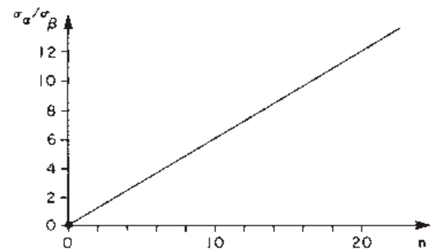

Fig. 7. Ratio $\sigma_\alpha/\sigma_\beta$ for different number of traverse legs.

The formula (19) is shown in a graphical form in Figure 7. For example, in a traverse consisting of 11 stations (10 legs) the same positional error $\sigma_{Y_{11}}$ will be obtained with either angles or azimuths if $\sigma_\alpha/\sigma_\beta$ = 6.2 It means that much less accurate gyro measurements will give the same positional accuracy as the much more precise angle measurements.

With present technology the gyro azimuth may be determined with a standard deviation ranging from 3″ when using sophisticated gyrotheodolites and time-consuming procedures to about 20″ when using small gyro attachments and comparatively simple techniques.

A standard deviation of 1.5″ for angle measurements may be easily obtained if systematic errors are taken care of, using a common 1″ theodolite and repeating the measurements in 4 to 6 sets.

Generally, however, the gyro azimuth determination is still more cumbersome and time-consuming than turning an angle in four sets. Because of rapid technological progress, this statement may not be valid in the near future.

In the diagram in Figure 8 a comparison of error propagation is given for traverses of different lengths up to 20 km with traverse legs equal to 500 m, and with angle or gyro azimuth measurements, or with a combination of both when using a gyrotheodolite at every 5th station. As can be seen from the diagram, the use of the gyro is not really practical for traverses shorter than 3 km (in this particular example) or, generally, for traverses with less than six legs. When the number of stations increases, then the use of gyro instruments improves the results considerably, even if the gyro measurements are only made at every 5th station of the traverse.

One could go on forever trying to analyze all other possible combinations of angles and gyro azimuths. More examples of combinations of this nature are given by *Ashkenazi* [1975].

Influence of atmospheric refraction and other sources of errors

The above discussion deals only with the propagation of random errors that could be estimated *a priori* in a comparatively easy way.

Much more dangerous are those sources of errors that may produce a systematic deflection of the underground open traverse. Some errors of this type may be produced by negligence or a faulty application of corrections to field observations. One example is neglecting to apply corrections arising from the deflections of the vertical. This is briefly discussed by *Richardus* [1974] with a more detailed account given by *Teskey* [1979], which includes a description of a simple new method that was developed at the University of New Brunswick for the determination of differences of the deflections of the vertical for engineering projects.

One of the most dangerous and difficult to discover sources of systematic errors is the influence of atmospheric refraction, and only this source of error will be discussed below.

Because the center portion of the tunnel is occupied by the transportation and construction machinery, underground traverses are usually established along and close to one wall of the driven tunnel. Heat transfer from the rocks surrounding the tunnel may produce a considerable gradient of temperature dT/dY near the wall in the direction perpendicular to the lines of sight of the control traverse. Some theoretical considerations on this subject are given by *Mendel* [1976].

Assuming, for example, that a uniform horizontal temperature gradient dT/dY persists over the whole length S of the optical line of sight of a traverse leg then the line will be curved and the point of sighting will be deflected by angle γ of lateral refraction which may be calculated [*Blachut et al* 1979] by the approximate formula

$$\gamma'' = 8''.0 \; \frac{P.S}{T^2} \left(\frac{dT}{dY}\right) \tag{20}$$

where P is the barometric pressure in millibars,

T is the temperature in Kelvin ($T = 273.15 + t°C$) and dT/dY is measured horizontally at right angles to the line of sight.

A constant gradient of only 0.01°C/m, if it persists over a distance of 1000 m, will cause a lateral refraction of 0.9″ for average atmospheric conditions.

If a straight traverse is measured (Figure 9) with angles $\beta = 180°$, then the constant refraction will produce errors of $\Delta\beta = 2\gamma$ at each station, bending the whole traverse along a circular curve and deflecting the last point by the amount

$$\Delta Y_n = \frac{\gamma_0 S_0}{\rho''} = \frac{8'' S_0^2 P}{T^2 \rho''} \left(\frac{dT}{dY}\right) \tag{21}$$

where S_0 is the length of the whole traverse, and

$$\rho'' = 206265''$$

If all the distances in the traverse are approximately the same and equal s, then S_0 in (19) may be replaced by $n.s$ and the equation may be rewritten in the form

$$\Delta Y_{n(\beta)} = n^2 s^2 \; \frac{8''}{T^2 \rho''} \left(\frac{dT}{dY}\right) \tag{22}$$

where n is the number of traverse legs.

A different deflection $\Delta Y_{n(\alpha)}$ will take place if gyro azimuths are measured at each station of the same traverse. Then, each individual azimuth α_i will be deflected by $\gamma = \dfrac{8'' PS_i}{T^2 \rho''} \dfrac{dT}{dY}$ and the deflection of the end point will be calculated from

$$\Delta Y_{n(\alpha)} = \frac{8'' P}{T^2 \rho''} \quad (s_1^2 + s_2^2 + \ldots + s_n^2) \; \frac{dT}{dY} = ns^2 \; \frac{8'' P}{T^2 \rho''} \left(\frac{dT}{dY}\right) \tag{23}$$

if $s_1 = s_2 = \ldots = s_n = s$

From (22) and (23) it can clearly be seen that the influence of the refraction on the traverse measured with a gyro instrument is n times smaller than in the case of angle measurements.

Let us take, for example, a straight traverse 6 km long with distances $s = 600$ m, $n = 10$ and $dT/dY = 0.02$°C/m at the temperature $T = 300$°K and $P = 1000$ mb. If only angles are measured, then the last point of the traverse will be deflected by $\Delta Y_6 = 112$ mm. If the same traverse is measured with a gyrotheodolite at each station then the systematic deflection of the traverse will only cause $112/6 = 19$ mm shift of the last point.

There is little information available on the expected values of the gradient of temperature in tunnels during their construction. It depends on many factors, such as

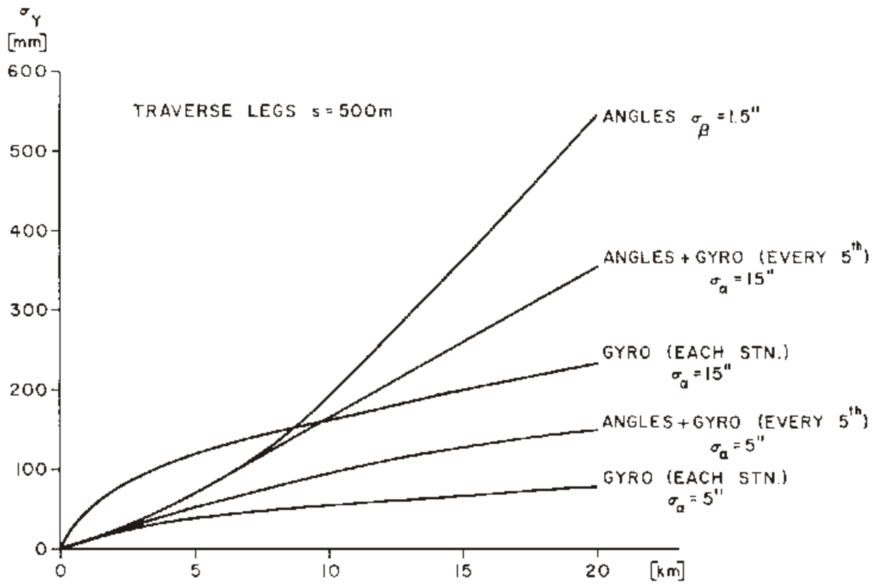

Fig. 8. Error propagation in open traverse with angle and/or gyro measurements (traverse legs s = 500 m).

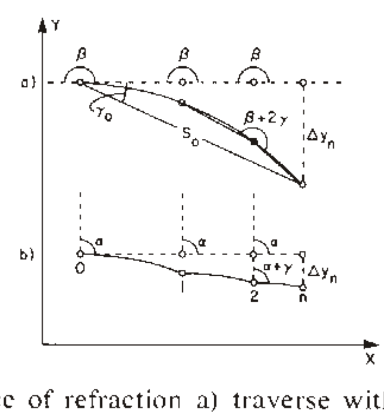

Fig. 9. Influence of refraction a) traverse with angles
b) traverse with gyro azimuths.

the type of rock, depth of the tunnel and the type of ventilation system. More research is needed. Nevertheless, several authors in describing case histories of tunneling surveys have observed some systematic deflections of traverses in their projects and have anticipated the influence of refraction.

Theoretically, one should expect a symmetrical distribution of the isolines of dT/dY in respect to the center line of the tunnel. Therefore, the control traverse should be established as closely as possible to the center line and as far as possible from the walls of the tunnel. Unfortunately this is usually impossible for the reasons mentioned previously. Another way of decreasing the influence of refraction is to run the traverse in a long zig-zag manner, placing the points on both sides of the tunnel. This may also be difficult due to many obstacles in the tunnel when crossing from side to side. Probably the easiest way is to run a double traverse on both sides of the tunnel with the same off-set distance from the center line. In any case, the underground control surveys should always be checked by at least two independent traverses. Therefore, one should try to run, for instance, the construction traverse on one side of the tunnel and the higher order checking traverse on the other side. If the theorem on the symmetrical distribution of the gradients of temperature is valid, then both traverses should be deflected in opposite directions. This means that a mean value from both traverses for setting out the center line of the tunnel should be almost free of refraction error. In this case it would be a mistake to correct the position of the last point of the construction traverse according to the results obtained only from the higher order traverse.

Therefore, if the use of the gyrotheodolite at each station is uneconomical, then frequent gyro checks should be made, say, at every 4th or 5th station, at least in the highest order control traverse.

References and bibliography

Ashkenazi, V. and R. J. Gough. Simulation games with a long tunnel, First Venezualan *Symposium on Geodesy*, Maracaibo, December 1975.

Blachut, T. J., A. Chrzanowski and J. Saastamoinen. *Urban Surveying and Mapping*, Springer-Verlag, N.Y., 1979.

Chrzanowski, A., E. Derenyi and P. Wilson. Underground survey measurements — Research for progress, *The Canadian Mining and Metallurgical Bulletin*, June 1967.

Chrzanowski, A. New techniques in mine orientation surveys, *The Canadian Surveyor*, Vol. 24, No. 1, March 1970;

Design and error analysis of surveying projects, Lecture Notes No. 47, University of New Brunswick, 1977.

Mendel, W. Refraktionsmodelle für Ingenieurmessungen unter besonderer Berücksichtigung des Stollenbaues, *VIIth International Course on Precision Engineering Surveys*, Technical University, Darmstadt, October 1976.

Posch, M. Survey work in constructing the Arlberg road tunnel, *Wild Reporter*, No. 16, 1980.

Richardus, P. *Project Surveying*, North-Holland Publishing Company, Amsterdam, 1974.

Rinner, K. Über die zweckmässige Anlage von Tunnelnetzen, *VIIth International Course on Precision Engineering Surveys*, Technical University, Darmstadt, October 1976.

Teskey, W. *Geodetic aspects of engineering surveys requiring high accuracy*, Technical Report No. 65, Department of Surveying Engineering, University of New Brunswick, 1979.

Wassermann, W. Underground survey procedures, *New Zealand Surveyor*, March 1967.